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Tugas II

▶ Soal Latihan 2.1

1. (a). $(4-3i)+(-8+2i)$

$= -4-i$

(d). $(2-i)(-3+2i)(5-4i)$

$= (-6+4i+3i-2i^2)(5-4i)$

$= (-6+7i+2)(5-4i)$

$= (-4+7i)(5-4i)$

$= (-20+16i+35i-28i^2)$

$= (8+51i)$

(g) $\frac{2-3i}{4-i} \rightarrow \frac{2-3i}{4-i}\frac{4+i}{4+i}$

$= \frac{8+2i-12i-3i^2}{16-i^2}$

$= \frac{11-10i}{17}$

$= \frac{11}{17} - \frac{10i}{17}$

(h) $\frac{5+5i}{3-4i} + \frac{20}{4+3i}$

$= \frac{(5+5i)(4+3i)+20(3-4i)}{(3-4i)(4+3i)}$

$= \frac{20+15i+20i+15i^2+60-80i}{12+9i-16i-12i^2}$

$= \frac{65-45i}{24-7i}\cdot\frac{(24+7i)}{(24+7i)}$

$= \frac{1560+455i-1080i-315i^2}{576-49i^2}$

$= \frac{1875-625i}{625}$

$= 3-i$

(k) $\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2}$

$= \frac{(6-4i+3i-2i^2)(1+2i)}{1-2i+i^2}$

$= \frac{(8-i)(1+2i)}{-2i}$

$= \frac{8+16i-i-2i^2}{-2i}$

$= \frac{10+15i}{-2i}\cdot\frac{i}{i}$

$= \frac{10i+15i^2}{-2i^2}$

$= \frac{10i-15}{2}$

$= -\frac{15}{2} + \frac{10}{2}i$

(l). $(-1+2i)^2\left(\frac{4}{1-i}+\frac{2-i}{1+i}\right)$

$= (1-4i+4i^2)\left(\frac{4(1+i)+(2-i)(1-i)}{(1-i)(1+i)}\right)$

$= (-3-4i)\left(\frac{4+4i+2-2i-i+i^2}{1-i^2}\right)$

$= (-3-4i)\left(\frac{5+i}{2}\right)$

$= \frac{-15-3i-20i-4i^2}{2}$

$= \frac{-11-23i}{2}$

$= -\frac{11}{2} - \frac{23}{2}i$

2). Diket = $z_1 = 1-i$ $\quad z_3 = \sqrt{3}-2i$

$z_2 = -2+4i$

Ditanya = (g). $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

Jawab :

(g). $\left|\frac{(1-i)+(-2+4i)+1}{(1-i)-(-2+4i)+i}\right|$

$= \left|\frac{3i}{3+4i}\right|$

$= \left|\frac{3i}{3+4i}\cdot\frac{(3-4i)}{(3-4i)}\right|$

$= \left|\frac{9i-12i^2}{9-16i^2}\right|$

$= \left|\frac{12+9i}{25}\right|$

$= \left| \frac{12}{25} + \frac{9}{25} i \right|$

$= \sqrt{\left(\frac{12}{25}\right)^2 + \left(\frac{9}{25}\right)^2}$

$= \sqrt{\frac{144+81}{625}}$

$= \sqrt{\frac{225}{625}}$

$= \frac{15}{25}$

$= \frac{3}{5}$ //

3. Bil kompleks z yang memenuhi $z^2 + (-3+2i)z + 5 - i = 0$ adalah ...

↳ $\underbrace{1}_{a} z^2 + \underbrace{(-3+2i)}_{b} z + \underbrace{5-i}_{c} = 0$

$a = 1 \; ; \; b = -3+2i \; ; \; c = 5-i$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-3+2i) \pm \sqrt{(-3+2i)^2 - 4(1)(5-i)}}{2(1)}$

$= \frac{3-2i \pm \sqrt{9+4i^2-12i-20+4i}}{2}$

$= \frac{3-2i \pm \sqrt{-15-8i}}{2}$

misal : $z = \sqrt{-15-8i}$

dengan z adalah bilangan kompleks $z = a+bi$

$z^2 = -15-8i = a^2 + 2abi - b^2$

$a^2 - b^2 + 2ab\,i = -15 - 8i$

kita dapatkan : $\begin{cases} a^2 - b^2 = -15 \\ 2ab = -8 \end{cases}$

$b = -\frac{4}{a}$

↳ $a^2 - \left(-\frac{4}{a}\right)^2 = -15$

$a^2 - \frac{16}{a^2} = -15$ $\times a^2$

$a^4 - 16 = -15a^2$

$a^4 + 15a^2 - 16 = 0$

$(a^2+16)(a^2-1) = 0$

$a^2 + 16 = 0$ $\qquad a^2 = 1$

$a^2 = -16$ (TM) $\qquad a = 1 \; V \; a = -1$

$a = 1 \rightarrow b = -4$ & $a = -1 \rightarrow b = 4$

$z = \pm 1 - 4i$

$x_1, x_2 = \frac{3-2i \pm z}{2}$

$= \frac{3-2i \pm 1-4i}{2}$

$x_1 = \frac{3-2i+1-4i}{2}$ $\qquad x_2 = \frac{3-2i-(1-4i)}{2}$

$= \frac{4-6i}{2}$ $\qquad = \frac{3-2i+4i-1}{2}$

$= 2-3i$ // $\qquad = \frac{2+2i}{2}$

$= 1+i$ //

Jawab : $(2-3i)$ atau $(1+i)$ //

4). Diketahui : $z_1 = 2+i \qquad z_2 = 3-2i$

Ditanya = $\left| \frac{z_1 + 2z_2 - 5 - i}{2z_1 - z_2 + 3 - i} \right|^2$

Jawab :

$\left| \frac{2+i+2(3-2i)-5-i}{2(2+i)-(3-2i)+3-i} \right|^2$

$= \left| \frac{3-4i}{4+3i} \right|^2$

$= \left| \frac{(3-4i)(4-3i)}{(4+3i)(4-3i)} \right|^2$

$= \left| \frac{12-9i-16i+12i^2}{16-9i^2} \right|^2$

$= \left| \frac{-25i}{25} \right|^2$

$= |-i|^2$

$= \left(\sqrt{(-1)^2}\right)^2$

$= (\sqrt{1})^2$

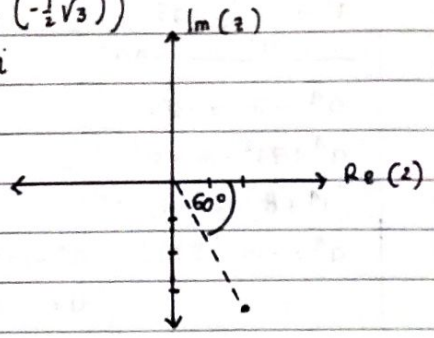
$= 1$ //

▶ Soal Latihan 2.2

2. (b). $z = 4(\cos 300^\circ + i \sin 300^\circ)$

$= 4\left(\frac{1}{2} + i\left(-\frac{1}{2}\sqrt{3}\right)\right)$

$= 2 - 2\sqrt{3}\,i$ //

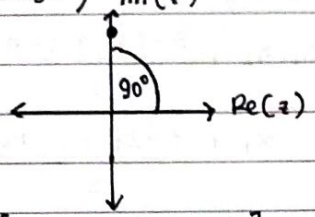


(c) $z = 12\ \text{Cis}\ 90^\circ$

$= 12\,(\cos 90^\circ + i \sin 90^\circ)$

$= 12\,(0 + i)$

$= 12\,i$



3. (c). $\frac{(8\,\text{cis}\,40^\circ)^3}{(2\,\text{cis}\,60^\circ)^2} = \frac{8^3\,[\cos 120^\circ + i \sin 120^\circ]}{2^2\,[\cos 120^\circ + i \sin 120^\circ]}$

$= \frac{(2^3)^3}{2^2}$

$= \frac{2^9}{2^2}$

$= 2^7$

$= 128$ //

(e). $\left(\frac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}\right)^{10} = \left(\frac{(1+\sqrt{3}\,i)(1+\sqrt{3}\,i)}{(1-\sqrt{3}\,i)(1+\sqrt{3}\,i)}\right)^{10}$

$= \left(\frac{1 + 2\sqrt{3}\,i + 3i^2}{1 - 3i^2}\right)^{10}$

$= \left(\frac{-2 + 2\sqrt{3}\,i}{4}\right)^{10}$

$= \left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}\,i\right)^{10}$

$* \ r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\sqrt{3}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{1}$

$= 1$

$a = r\cos\theta$

$-\frac{1}{2} = 1 \cos\theta$

$b = r\sin\theta$

$\sin\theta = \frac{1}{2}\sqrt{3}$

$\theta = 120^\circ$

$z = (\cos 120^\circ + i \sin 120^\circ)^{10}$

$z = \cos 1200^\circ + i \sin 1200^\circ$

$= \cos(1080^\circ + 120^\circ) + i \sin(1080^\circ + 120^\circ)$

$= \cos 120^\circ + i \sin 120^\circ$

$= -\frac{1}{2} + \frac{1}{2}\sqrt{3}\,i$ //

5). (d). $w = (2\sqrt{3} - 2i)^{1/2}$

$* \ r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$

$= \sqrt{12 + 4}$

$= \sqrt{16}$

$= 4$

$* \ a = r\cos\theta$

$2\sqrt{3} = 4\cos\theta$

$\cos\theta = \frac{1}{2}\sqrt{3}$

$b = r\sin\theta$

$-2 = 4\sin\theta$ $\quad \theta = 330^\circ$

$\sin\theta = -\frac{1}{2}$

$z = 4\,(\cos 330^\circ + i \sin 330^\circ)$

$w = z^{1/2} = 4^{1/2}\left(\cos\left(\frac{\theta + 2k\pi}{n}\right) + i \sin\left(\frac{\theta + 2k\pi}{n}\right)\right)$

▶ Pertambahan Sudut $= \frac{360^\circ}{n} = \frac{360^\circ}{2} = 180^\circ$

$w_1 = 2\left(\cos\frac{330}{2} + i \sin\frac{330}{2}\right)$

$= 2\,(\cos 165^\circ + i \sin 165^\circ)$

$= 2\,(-0{,}96 + 0{,}258\,i)$

$= -1{,}931 + 0{,}517\,i$ //

$w_2 = 2\ \text{Cis}\ 345^\circ$

$= 2\,(\cos 345^\circ + i \sin 345^\circ)$

$= 2\,(0{,}96 - 0{,}258\,i)$

$= 1{,}931 - 0{,}517\,i$ //

Jawab : $w_1 = -1{,}931 + 0{,}517\,i$

$w_2 = 1{,}931 - 0{,}517\,i$

(f). $w = (-4 + 4i)^{1/5}$

$* \ r = \sqrt{(-4)^2 + 4^2}$

$= \sqrt{16 + 16}$

$= \sqrt{2 \cdot 16}$

$= 4\sqrt{2}$

$* \ a = r\cos\theta$

$-4 = 4\sqrt{2}\cos\theta$

$-\frac{1}{2}\sqrt{2} = \cos\theta$

$\theta = 135^\circ$

$b = r\sin\theta$

$4 = 4\sqrt{2}\sin\theta$

$\sin\theta = \frac{1}{2}\sqrt{2}$

$z = 4\sqrt{2}\,(\text{cis}\ 135^\circ)$

$w = z^{1/5} = (2^{5/2})^{1/5}\left[\text{cis}\ \frac{135 + 2k\pi}{5}\right]$

$= \sqrt{2}\left[\text{cis}\ \frac{135 + 2k\pi}{5}\right]$

$w_1 = \sqrt{2}\left[\cos\frac{135}{5} + i \sin\frac{135}{5}\right]$

$= \sqrt{2}\,[\cos 27^\circ + i \sin 27^\circ]$

$= \sqrt{2}\,(0{,}891 + 0{,}454\,i)$

$= 1{,}26 + 0{,}642\,i$

* Pertambahan Sudut = $\frac{360^\circ}{n} = \frac{360^\circ}{5} = 72^\circ$

$W_2 = \sqrt{2}\,(\cos 99^\circ + i \sin 99^\circ)$

$= \sqrt{2}\,(-0{,}15643 + i \cdot 0{,}98768)$

$= -0{,}22123 + 1{,}3968\,i$

$W_3 = \sqrt{2}\,(\cos 171^\circ + i \sin 171^\circ)$

$= \sqrt{2}\,(-0{,}17364 + 0{,}156434\,i)$

$= -0{,}245575 + 0{,}221231\,i$

$W_4 = \sqrt{2}\,(\cos 243^\circ + i \sin 243^\circ)$

$= \sqrt{2}\,(-0{,}454 + i\,(-0{,}891))$

$= -0{,}642 + (-1{,}26)i$

$= -0{,}642 - 1{,}26\,i$

$W_5 = \sqrt{2}\,(\cos 315^\circ + i \sin 315^\circ)$

$= \sqrt{2}\,(\frac{1}{2}\sqrt{2} - i \cdot \frac{1}{2}\sqrt{2})$

$= 1 - i$

Jawaban : $W_1 = 1{,}26 + 0{,}642\,i$

$W_2 = -0{,}22123 + 1{,}3968\,i$

$W_3 = -0{,}245575 + 0{,}221231\,i$

$W_4 = -0{,}642 - 1{,}26\,i$

$W_5 = 1 - i$ //

6. (b). $z^2 + (-3+2i)z + 5 - i = 0$

$\underset{a}{1}\,z^2 + \underset{b}{(-3+2i)}\,z + \underset{c}{5-i} = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-3+2i) \pm \sqrt{(-3+2i)^2 - 4(1)(5-i)}}{2(1)}$

$= \frac{3-2i \pm \sqrt{9+4i^2-12i-20+4i}}{2}$

$= \frac{3-2i \pm \sqrt{-15-8i}}{2}$

-> Misal : $z = \sqrt{-15-8i}$

dengan z adalah bilangan kompleks $z = a+bi$

$z^2 = -15-8i = a^2 - b^2 + 2abi$

Didapatkan : $\begin{cases} a^2 - b^2 = -15 \\ 2ab = -8 \end{cases}$

$\rightarrow b = -\frac{4}{a}$

$a^2 - b^2 = -15$

$a^2 - \left(-\frac{4}{a}\right)^2 = -15$

$a^2 - \frac{16}{a^2} = -15 \quad \times a^2$

$a^4 - 16 = -15a^2$

$a^4 + 15a^2 - 16 = 0$

$(a+16)(a^2-1) = 0$

$a = -16$ (TM) $\quad a^2 - 1 = 0$

$a = 1 \vee a = -1$

$a = 1 \rightarrow b = -4$; $a = -1 \rightarrow b = 4$

$z = \pm 1 - 4i$

$x_1, x_2 = \frac{3-2i^* \pm 1-4i}{2}$

$x_1 = \frac{3-2i+1-4i}{2}$	$x_2 = \frac{3-2i-(1-4i)}{2}$
$= \frac{4-6i}{2}$	$= \frac{2+2i}{2}$
$= 2-3i$	$= 1+i$

Jawab : $(2-3i)$ atau $(1+i)$ //

(e). $z^4 + z^2 + 1 = 0$

misal : $p = z^2 \rightarrow p^2 + p + 1 = 0$

$p_1, p_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2}$

$= \frac{-1 \pm \sqrt{-3}}{2}$

$= \frac{-1 \pm i\sqrt{3}}{2}$

$p_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$; $p_2 = -\frac{1}{2} - \frac{i\sqrt{3}}{2}i$

$p = z^2$, dengan $z = a+bi$

maka $z^2 = a^2 - b^2 + 2abi$

① $a^2 - b^2 + 2abi = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

• Didapatkan : $\begin{cases} a^2 - b^2 = -\frac{1}{2} \\ 2ab = \frac{\sqrt{3}}{2} \rightarrow b = \frac{\sqrt{3}}{4a} \end{cases}$

→ $a^2 - b^2 = -\frac{1}{2}$

$a^2 - \left(\frac{\sqrt{3}}{4a}\right)^2 = -\frac{1}{2}$

$a^2 - \frac{3}{16a^2} = -\frac{1}{2}$ $\times 16a^2$

$16a^4 - 3 = -8a^2$

$16a^4 + 8a^2 - 3 = 0$

$(4a^2 + 3)(4a^2 - 1) = 0$

$4a^2 + 3 = 0$ $\quad$ $4a^2 - 1 = 0$

$4a^2 = -3$ $\quad$ $4a^2 = 1$

$a^2 = -\frac{3}{4}$ (TM) $\quad$ $a^2 = \frac{1}{4}$

$a = \frac{1}{2}$ V $a = -\frac{1}{2}$

$a = \frac{1}{2} \longrightarrow b = \frac{1}{2}\sqrt{3}$, $a = -\frac{1}{2} \rightarrow b = -\frac{1}{2}\sqrt{3}$

$z_1 = \frac{1}{2} + \frac{1}{2}\sqrt{3}\,i$

$z_2 = -\frac{1}{2} - \frac{1}{2}\sqrt{3}\,i$

② $a^2 - b^2 + 2ab\,i = -\frac{1}{2} - \frac{\sqrt{3}}{2}\,i$

Didapatkan : $\begin{cases} a^2 - b^2 = -\frac{1}{2} \\ 2ab = -\frac{\sqrt{3}}{2} \rightarrow b = \frac{-\sqrt{3}}{4a} \end{cases}$

→ $a^2 - b^2 = -\frac{1}{2}$

$a^2 - \left(\frac{-\sqrt{3}}{4a}\right)^2 = -\frac{1}{2}$

$a^2 - \frac{3}{16a^2} = -\frac{1}{2}$ $\times 16a^2$

$16a^4 - 3 = -8a^2$

$16a^4 + 8a^2 - 3 = 0$

$(4a^2 + 3)(4a^2 - 1) = 0$

$4a^2 + 3 = 0$ $\quad$ $4a^2 - 1 = 0$

$4a^2 = -3$ $\quad$ $4a^2 = 1$

$a^2 = -\frac{3}{4}$ (TM) $\quad$ $a^2 = \frac{1}{4}$

$a = -\frac{1}{2}$ V $a = \frac{1}{2}$

$a = -\frac{1}{2} \longrightarrow b = \frac{\sqrt{3}}{2}$; $a = \frac{1}{2} \rightarrow b = -\frac{\sqrt{3}}{2}$

$z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$; $z_4 = \frac{1}{2} - \frac{\sqrt{3}}{2}\,i$

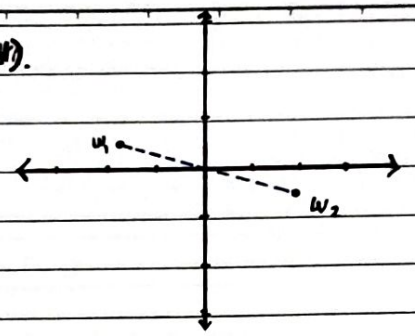
Jawab : •> $z_1 = \frac{1}{2} + \frac{1}{2}\sqrt{3}\,i$

•> $z_2 = -\frac{1}{2} - \frac{1}{2}\sqrt{3}\,i$

•> $z_3 = -\frac{1}{2} + \frac{1}{2}\sqrt{3}\,i$

•> $z_4 = \frac{1}{2} - \frac{1}{2}\sqrt{3}\,i$ //

5). d).



e).